Barem de corectare OLM 2019 Clasa a X-a

P1 – autor Traian Tămâian (GM 11/2018)

$\left(1-5^x\right)\left(2^x-3^x-6^x\right)=0$	3p
$5^x=1 \Rightarrow x=0$	1p
$6^x+3^x=2^x \Rightarrow 3^x+\left(\frac{3}{2}\right)^x=1 \Rightarrow x=-1$; soluție unică -1	3p

P2

$E(x)=\frac{1}{\left(\sqrt{x}-\sqrt{y}\right)^2}\frac{x+y}{xy}+2\frac{\sqrt{y}-\sqrt{x}}{\sqrt{xy}}\frac{1}{\left(\sqrt{x}-\sqrt{y}\right)^3}=$	2p
$E(x)=\frac{1}{\left(\sqrt{x}-\sqrt{y}\right)^2}\frac{1}{xy}\left(x+y-2\sqrt{xy}\right)=\frac{1}{xy}$	3p
$E\left(2^{4-\sqrt{7}},2^{4+\sqrt{7}}\right)=\frac{1}{2^8}=\frac{1}{256}$	2p

P3

a) $f(x)=\log_3 x\cdot\log_9 x+...+\log_{3^n} x\cdot\log_{3^{n+1}} x \Leftrightarrow f(x)=\frac{n}{n+1}\log_3{}^2 x$	1p
$x_1,x_2\in(0,1)$, $\log_3{}^2 x_1=\log_3{}^2 x_2 \Rightarrow x_1=x_2$ sau $x_1=\frac{1}{x_2}$, imposibil, deci funcția este injectivă.	1p
Pentru $y<0$ ecuația $\frac{n}{n+1}\log_3{}^2 x=y$ nu are soluţii, deci funcția nu este surjectivă.	1p
b) Inecuația devine $\log_3{}^2 x\le\left(\frac{n+1}{n}\right)^2$ sau $-\frac{n+1}{n}\le\log_3 x\le\frac{n+1}{n}$	1p
Cum $\log_3 x<0 \Rightarrow -\frac{n+1}{n}\le\log_3 x<0$	1p
$x\in\left[3^{-\frac{n+1}{n}},1\right)$; din condiția $3^{-\frac{n+1}{n}}\le 3^{-\frac{8}{7}} \Rightarrow n\le 7$	1p
$n\in\{5,6,7\}$	1p

P4

a) $\lvert z_k\rvert=1 \Rightarrow \overline{z_k}=\frac{1}{z_k}, k\in\{1,2,3\}$; atunci $\overline{z_1}+\overline{z_2}+\overline{z_3}=1$ și $z_1z_2+z_2z_3+z_3z_1=z_1z_2z_3$	2p
$1=\left(z_1+z_2+z_3\right)^2 \Rightarrow z_1{}^2+z_2{}^2+z_3{}^2-1=-2\left(z_1z_2+z_2z_3+z_1z_3\right)=-2z_1z_2z_3$, deci $\left\lvert z_1{}^2+z_2{}^2+z_3{}^2-1\right\rvert=2$	1p
b) Din $z_1z_2+z_2z_3+z_3z_1=z_1z_2z_3$, folosind prima condiție, se obține $\left(1-z_3\right)\left(z_3+z_1z_2\right)=0$.	2p
Dacă $z_3=1$, atunci $z_2=-z_1$, deci $z=1\in R$	1p
Dacă $z_3+z_1z_2=0$, atunci $\left(z_1-1\right)\left(z_2-1\right)=0$, deci $z=1\in R$	1p